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The Harmonic Interchangeability Variance of Cauchy Constant's Value: A and B

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Abstract

The strong relationship of refractive index of a material with wavelengths dates back to more than a century especially where Sellmeier's equation was invented by Wolfgang Sellmeier in 1871 and then developed by Cauchy for visible region of electromagnetic waves. In this work, we studied the interchangeability of Cauchy constants A and B for a glass prism by using inert gas light sources such as He and Ne and metal vapor light sources as Hg, Cd, HgCd, and Na. Cauchy formula represents this dependence where A and B are Cauchy constants and their value are harmonically interchangeable. We investigated that each light sources have a different value of A and B and does not follow the same pattern of graphical representation. From this result, one can notice that Cauchy constants have a crucial role on this dependence of refractive indices on the wavelength of the light sources. The novelty of this work is the combination of Geometrical Optics and Wave Optics, then formulating a new equation, which is exactly geometrical measurements.

I. Introduction

Inert gases or any other fluorescent gases could have different spectrums [1]; we mean that the visible region of electromagnetic waves (EMW). In this region, the electrons transition from the higher levels of energy to the second level which has the principal number n=2 [2]. The sets of wavelengths are different from one source to the other due to their atomic number and their interaction with electromagnetic waves [3].

When a polychromatic light passes through a prism, it bends toward the normal of the surface and the prism separates the light to its original color by an angle depending on their wavelengths [4]. Regarding to the wave optics, gratings can be used to separate polychromatic light as well [5]. From the use of both apparatus, the combination between geometrical and wave optics, minimum angle of deviation of refraction and the wavelength of separated lines through a spectrometer can be measured. Researchers have looked into these combinations in some diverse perspective, which are different from this work [6, 7, 8, 9, 10].

The significant of these two procedures is to show how the refractive indices of a material vary with the wavelength of the light [11, 12]. More importantly, focusing on Cauchy formula. Since the formula is in the form of series, for experimental approach, we will aim at the lower orders of the series [12, 13]. Briefly, the research will calculate Cauchy constants A and B for different light sources of polychromatic light as it has been compared for Sellmeier [14]. For this purpose, the method and materials for this work is shown in the following sections. In addition, the relation between Geometrical Optics and Wave Optics are explained and formulating new equation.

II. Spectral Line for Polychromatic Sources

Normally, electrons in atoms situated at the lowest energy level. The electrons can be excited to higher energy levels by absorbing energy; one can provides this energy by light, heat, or electrical discharge. The electrons from excited states will return to the lower energy levels, then return to the ground state. They emit energy in the form of light [15]. In this work, a high voltage electrical discharge passed through the inert gases such as (He and Ne) and metal vapor lamps of, Cd, Hg, Hg-Cd and Na at low pressure, then the light is emitted when this light incident on a prism surface [12]. One can notice that the light is contained a set of specific wavelengths which is known as polychromatic light [2]. An emitted light has amount of energy exactly equal to the difference between the two energy levels [16]. In the other word, the amount of energy, which is lost by electron as it returns to the ground energy level, is equal to the energy of the emitted photon [2, 12].

Each individual element has different energy levels when the electrons return to their lower energy level they emit photons of different color, because each atom has different excited states for that reason it emits a several colors of light. The individual color, which emitted by an electron, is called line spectra [1, 15]. Planck's equation provides that we can observe only so wavelength or frequency values in the line spectra; this means that some values of energy are possible. Hence, the energy states of the atom are quantized, this means that those levels are allowed that have definite values of energy [12]. The spectrum of each element have many lines, human eye can only detect lines that are in the visible range (400-700) nm or near visible range [2, 17]. The series of observed spectra line is called Balmer Series, all the spectra lines in this series resulting from the transition of electrons form excited states to the second energy level $n_i = 2$.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \ldots\ldots\ldots\ldots\ldots (1)$$

Where $R = 1.097 \times 10^7\ m^{-1}$ is Rydberg constant, $n_i = 2$ and $n_f = (3,4,5,\ldots)$ they are positive constants [15]. The other lines cannot be observed. It observed that there are some wavelengths of the Lyman series in the emitted line spectra of the Mercury (Hg) lamp, which include the transition of electrons from higher energy level to energy level with $n_1 = 1$, it should be ultraviolet range [16]. Figure (1) shows the transitions of electrons from excited states to low energy states.

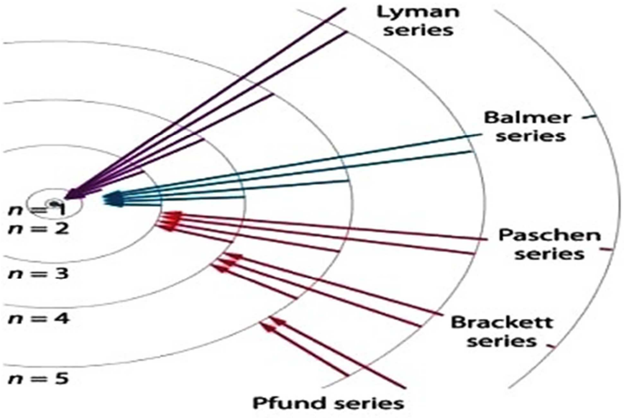

Figure 1: The transition of electrons from higher levels to lower level and emitting light [13].

III. Methods and Materials

The experimental part of this work is explained from both geometrical and physical points of view separately in the following:

A. Wavelength Measurement

In this work, we used two lamps of inert gases He and Ne and four metal vapor lamps, which are: Hg, Cd, Hg-Cd and Na and each lamp is made of different element, as it is clear from their name referring to its

chemical name in periodic table, in which, each lamp has different sets of spectral lines [16]. When the light of lamp passes through a grating, it therefore splits to their original color, and then each line has specific wavelength (color), which diffracted by different angle depending on its wavelength [19].
Knowing the number of lines of a grating ($l_m \equiv$ lines per millimeter)[1], and from that, line spacing d, the wavelength of the lines for each lamp can be found from the following equation (grating equation) [20]:

$$d \sin\theta = n\lambda \leftrightarrow \sin\theta = n l_m \lambda, \ldots\ldots\ldots (2)$$

Where n = 0, 1, 2, ⋯ is the order of the fringes (bright lines), λ is the wavelength of a line (color) and θ is the diffracted angle of the lines by the grating [21]. The recent paper shows two ways of wavelength measurement with homemade apparatus, which are diffraction by transmission and by reflection at a CD, or any compact discs [2]. We used a transmission grating and the wavelengths are measure to a great accuracy [22].

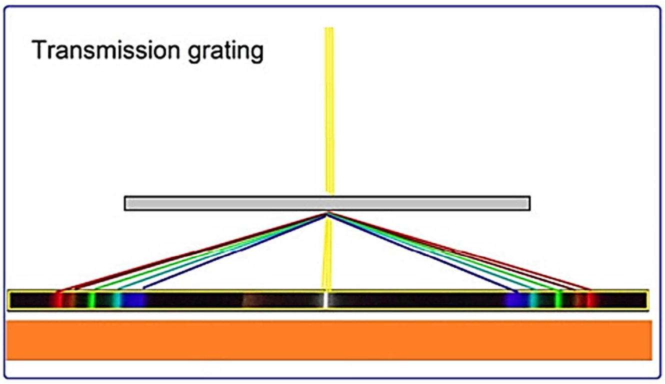

Figure 2: separation of polychromatic light by a grating [3]

B. Spectral Line Angle (Dispersion Angle of Atomic Spectral Line)

Generally, a transparent medium in which its refractive index depends on the wavelength of the light source that travels through the medium, which is called dispersive medium such as (transparent plastic, glass, water, and quartz). In the normal dispersion the long wavelength travel quicker than do short wavelengths [10]. This means refractive index of dispersive medium depends on the wavelengths of the light as shown in the figures (3). Each individual color will refract by different angle, because refractive index of a dispersive medium is a function of the wavelength, so the refraction angle depends on the wavelength [18]. We have used a triangular prism; however, the shape of the slits has no effect on the angles at which different wavelengths are diffracted [5].

The convenient formula to measure the refractive index of a transparent medium is calculated from finding the angle of minimum deviation $\delta_{\min}$ [16]. One can notice that where the angle is at minimum deviation, the emerging ray will have a symmetrically path through the prism. In the case of the symmetrically path of the ray through the prism, the refractive index is given as in equation (3) [2, 5, 23]. The point, here, is to find the angle of refraction of each line (wavelength), for this, using the equation of prism as in equation (3), we have calculated the minimum angle of deviation of each spectral line ($\delta_{\min}$).

$$n = \frac{\sin(\frac{1}{2}(\phi+\delta_{\min}))}{\sin(\phi/2)} \ldots\ldots\ldots\ldots\ldots (3)$$

where φ is the apex angle of the prism [14].

IV. Cauchy Formula of Refraction

Cauchy formula is an empirical formula to show the relationship between the index of refraction of materials and wavelengths of light for transparent materials [19]. The formula dates back to his work on spectral theory in his "Cauchy memoir" and it was developed in 1836 from Sellmeier's equation for a short-

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[1] We have used a grating with l_m= 1000 lines/mm.

range of wavelengths [20, 25]. It can be shown in an expansion series as in equation (4) [5]. The theory of light-matter interactions gives:

$$n = \sum_{i=1}^{k} a_i \lambda^{-2i} = a_0 + \frac{a_1}{\lambda^2} + \frac{a_2}{\lambda^4} + \cdots \ldots\ldots (4)$$

For simplification of experimental result, the higher terms are neglected, in general, it is seen in several references that a_0 = A, a_1 = B, a_2 = C and so on. The letters A, B and C are called Cauchy constants and they can be found experimentally for different materials [5]. The relation is seen in different notations as in [20]. The constants have physical meaning:

$$0 < \mathrm{C} < \mathrm{B} < 1 < \mathrm{A} \ldots\ldots\ldots (5)$$

The constants contains a deep detail regarding light-atom interactions, in which, the atomic number of the material is vital [14, 26]. The relation (4) clarifies the dispersion relation of materials, which is why sometimes it is called Cauchy dispersion formula. It can be explained with the concept of secondary waves that are produced by the induced oscillations of the bound charges [12]. When a beam of light propagates through a transparent medium (solid or liquid), the amount of lateral scattering is extremely small [20]. The scattered waves travelling in a lateral direction produce destructive interference. Cauchy formula has a significant advantage of choosing materials in thin films to reduce the reflection (i.e. strengthen antireflection) [27, 28]. As it is clear from the graphs, which is between refracted index (n) on y-axis and the invers of wavelength squared on x-axis, it gives a linear relationship. The intercept with y-axis is (B) and with y-axis is (A). They should have the same A and B value for each lamp, but they are depending on the same taken data. The dependence of the refractive index on the wavelength, can be expressed this dependence as [29]:

$$\frac{\mathrm{d}\delta_{\mathrm{min}}}{\mathrm{d}\lambda} = \frac{\mathrm{d}\delta_{\mathrm{min}}}{\mathrm{dn}} \cdot \frac{\mathrm{dn}}{\mathrm{d}\lambda} \ldots\ldots\ldots\ldots (6)$$

The first fraction of the right side dn/dλ of equation (6) depends on the states of the experiment; the second fraction is dispersion depends only on the material of the glass prism, which has shown by Hartmann dispersion as well [11, 21]. Bach has presented various dispersion formulas for glasses in general [22]. We see Cauchy's formula (equation (4)) for a particular wavelength is

$$n(\lambda) = A + \frac{B}{\lambda^2} \ldots\ldots\ldots\ldots\ldots (7)$$

We can obtain the dispersion of the substance by differentiation equation (4) with respect to the wavelength [16], then

$$\frac{\mathrm{dn}}{\mathrm{d}\lambda} = -2\frac{\mathrm{B}}{\lambda^3} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

V. Geometrical Optics verses Wave Optics

The combination between geometrical optics and wave optics from different perspective done by researchers inspired us to notice a new way of understanding of how a spectrum of a source is analyzed. The fact of the difference of the atomic property of light source affects on the individual shifting of angle of refraction in materials (e.g. prism) and angle of diffraction (e.g. grating) for the same sets of wavelengths.

For visible region, since the wavelengths are very small, it is allowed to neglect the higher terms of the wavelengths as we have shown in equation (7) from equation (4). We take geometrical optics verses wave optics interpretation in this subject as a new idea by equating both equations (3) and (7), throughout equation (2), we formulate the angle of diffraction (θ) as a function of the angle of minimum deviation (δmin), as θ = f (δ_{min}) so

$$\sin\theta = \sqrt{\frac{\mathrm{B}\sin(\phi/2)}{\sin((\delta_{\mathrm{M}}+\phi)/2) - \mathrm{A}\sin(\phi/2)}} \cdot \mathrm{nl}_{\mathrm{m}} \ldots\ldots\ldots (9)$$

Since A, B and φ are constants, it is seen that the angle of diffraction, nonlinearly, is inversely proportional to the angle of refraction (i.e. minimum deviation angle). Experimentally, the significance of this combination (i.e. equation 9) is seen in figure (4). In addition, the formulated equation (Eqn. (9)) is coincident with the results of the figures. Another strong side of this equation is that it can be used to determine the wavelengths of a light source by using prism instead of a grating, in other words from the refraction concept instead of diffraction concept as:

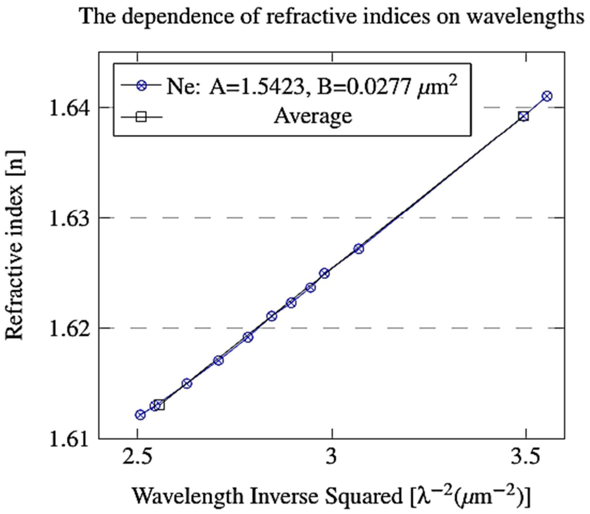
$$\lambda = \sqrt{\frac{B\sin(\phi/2)}{\sin((\delta_M+\phi)/2)-A\sin(\phi/2)}}\cdot n\ldots\ldots\ldots (10)$$

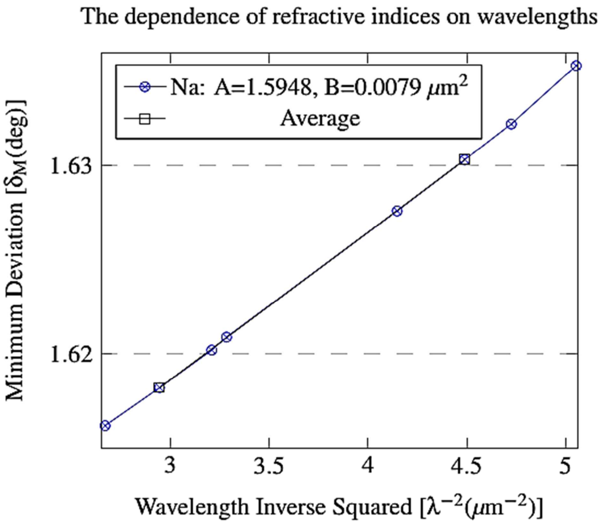
For a specific prism where A and B are known, the wavelength of a light source can be calculated through geometrical measurement without the interfere the wave nature of light. In other words, the wavelike part is on the left side and the geometrical part is on the right side in equation (10).

VI. Results and Discussion

The following graph charts illustrate interchangeability of Cauchy constants A and B for a given glass prism by using different inert gas and metal vapor light sources as it mentioned before.

Overall, the refractive index of a glass prism significantly increased with the reciprocal of the square of the wavelengths of the light sources, which means that the refractive index of a dispersive material is inversely proportional with the wavelength of the light.


The dependence of refractive indices on wavelengths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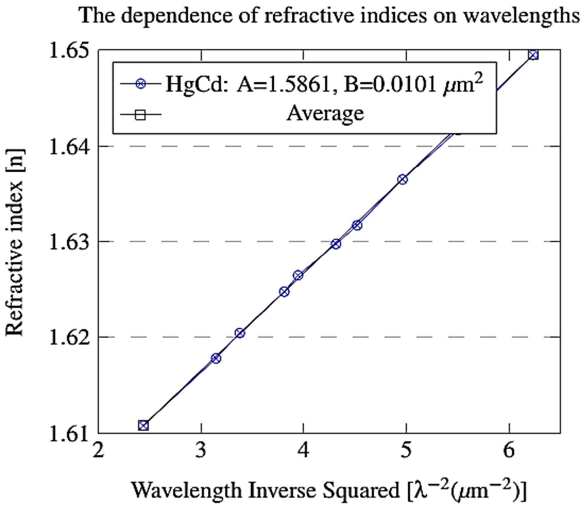
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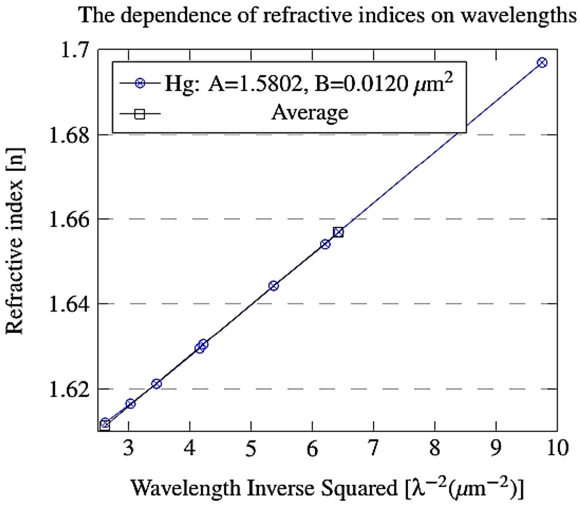
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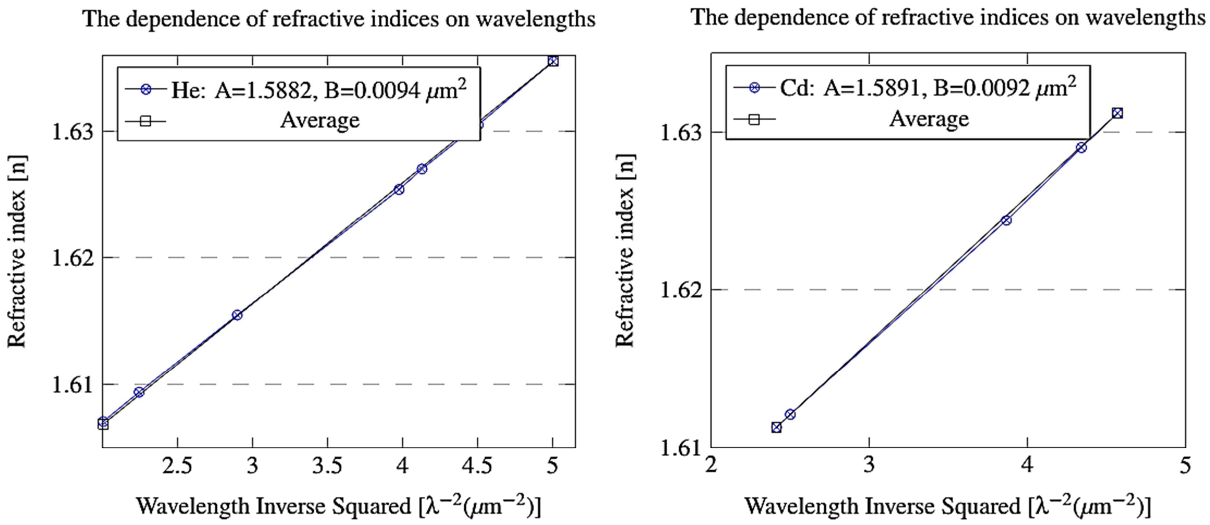

Figure (3): The dependence of refractive indices on wavelength for Ne, Na, HgCd, Hg, He and Cd light source.

This dependence is represented by Cauchy formula equation (7). Then we found those constants for the given glass prism for each light source. One can notice form figure (3) that the given glass prism has a slightly different values of *A* and B for a different light sources, as it is shown in table (1).

Table 1: Cauchy constants of a glass prism for different light sources

	Constants		
Source	A	B	R^2
Na	1.5948	0.0079	0.9998
Cd	1.5891	0.0092	0.9998
He	1.5882	0.0094	0.9998
Hg-Cd	1.5861	0.0101	0.9997
Hg	1.5802	0.0120	0.9999
Ne	1.5423	0.0277	0.9997

The most striking feature of these results is the values of and B, for each source. The value of A increased by decreasing B value, this means their relation is interchangeable. For example, the glass prism has A = 1.5423 and B = 0.0277 for the light source Ne lamp. On the other hand, Na lamp has A = 1.5948 and B = 0.0079. Cowan and Jose provides that the Cauchy constants are functions of wavelengths of the used light sources that came from the interaction of light source with atom [1, 30]. We have put the data for the source from the highest to the lowest in A, while it is opposite for B. For this calculation we have determined the coefficient of determination (R^2) [31], which evaluates how well the data are fitting a statistical model. It has a value between 0 and 1 [32]. A value 1 indicates that the data are fitting more precisely, while a value 0 illustrates that the line dose not appropriate the data at all it means there are an error in the date. It can be noticed that the values of R^2 for the data we have taken experimentally are more precise, all the values of A and B about $R^2 \approx 1$ as shown in the table (1) [31]. The least error in the coefficient of determination may refer to the simplicity of our instruments such as spectrometer including the efficiency of eyepiece and resolving power, in addition, the life time could have an impact on the accuracy of the results. Figure (4) shows the relation between geometrical optics, in particular, the minimum deviation angles and the diffraction angles of the wavelengths in wave optics.

Generally speaking, their relation is exponential. The diffraction angles of the wavelengths of the light sources that are used in this work are decreasing exponentially with the angles of minimum deviation. The most remarkable point is the line graph of the Ne light source, which its relation is not exactly exponential, in which the diffracted angles slightly declined linearly with the minimum deviation angles of the wavelengths of the Ne light source. The variance may refer to the high rate of number of line per spectral width as compared to the other light sources. In other words, Ne has short spectral width and many spectral lines. Regarding to the dependence of refractive indices on the wavelengths of the light sources, the relation is linearly increased for all the light sources. Neon line graph does not join the other light sources line graph, in which its line slightly sets upper from the other, as shown in the figures (4) and (5). These results may have a dramatic use in the interpretation or which are used to find Cauchy constants A and B have roles on the variance of the A and B values for the same sets of wavelengths. The variance of Cauchy constants for the same material (i.e. crown prism) is strictly seen in a source (Ne).

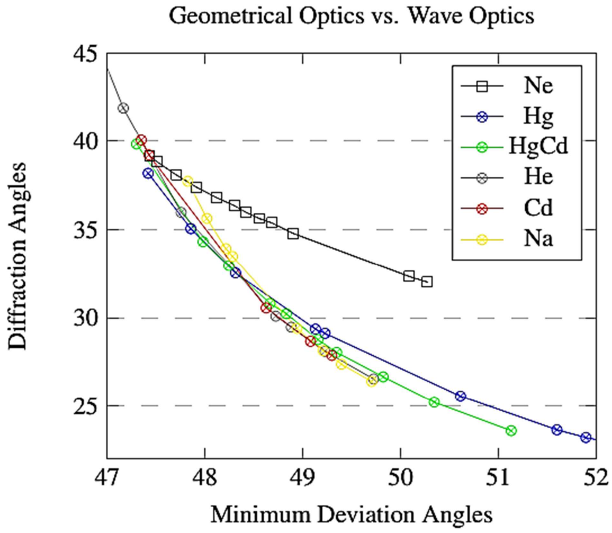

Figure 4: Relation between Geometrical and Wave optics.

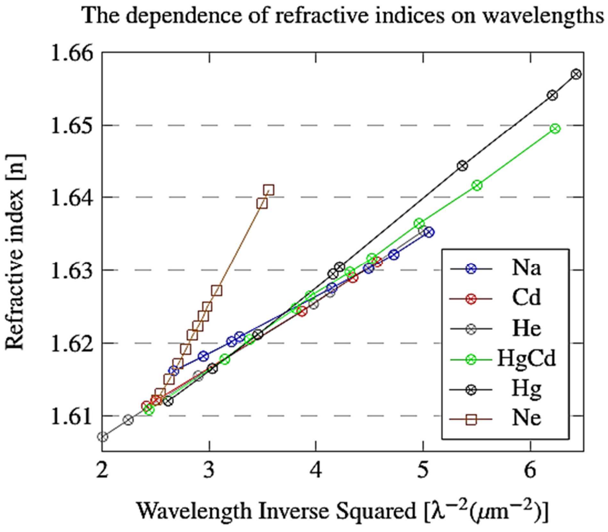

Figure 5: The dependence of refractive indices of wavelength for all the light sources.

VII. Conclusion

Nowadays, researchers and laboratorists attempt to obtain the best results from their equipment. The instruments are going to be advanced from day to day. However, some try to do the same procedure at home. We make benefit from the idea with a great accuracy

In this work, we focused on Cauchy formula in optics. The formula shows the dependence of refractive index on wavelengths. Besides of this dependence, we concluded that the sources which are used to find Cauchy constants A and B have roles on the variance of the A and B values for the same sets of wavelengths, the value of A and B have a crucial role on choosing an appropriate light source to interact with a specific medium to obtain more accuracy result. The variance of Cauchy constants for the same material (i.e. crown prism) is strictly seen in a source of short spectral width (eg. Ne).

Another look of this work is to combine geometrical optics and wave optics in an equation. By the end of this combination, we sum up with the fact that the direct calculation of the angles of refraction (Geometrical Optics) and diffraction (Wave Optics). Although the total refractive index of the prism remains constant, it harmonically confirms that the polychromatic light sources affect on the value of A and B for the same material. From this, we can say that Cauchy constants do not only depend on the wavelengths, but also on the light sources used in the experiment. In which it means that the light source do not give the same pattern

of refraction and diffraction for the same set of wavelength. In addition, we notice that each light source has a different set of wavelengths which is totally like a fingerprint of human, so that it can be used to recognize the type of source from the other. These results may have a dramatic use in the interpretation or analyzing spectrums from a source such as in astronomy for stars and galaxies.
A great finding in this circumstance, is that the wavelengths of any light source can be obtained from geometrical measurement without the entrance to the wavelike world of light.

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